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Marangoni–Bénard instability of a floating liquid layer with an internal, permeable, heated or cooled divider and two deformable open surfaces

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Abstract—The results of a linear stability analysis of a floating thin liquid layer with both sides open to air and internally split in two halves by a permeable, heated or cooled divider are presented. For the case of deformable open surfaces and varying thermal and mechanical divider characteristics the results refer to steady and overstable modes of convection, hence leading to surface waves. Results are also provided about the role of the Marangoni effect in exciting antisymmetrical/flexural and symmetrical/squeezing modes of vibration of the thin liquid layer.

1. INTRODUCTION

The onset of Marangoni–Bénard convection in a heated or cooled layer with a free surface open to ambient air has been the subject of many investigations, which have identified various mechanisms of instability [1–15]. However, if both surfaces of a floating layer are free and moreover deformable one may expect interaction of instability modes to occur at these two opposite surfaces, hence leading to new and genuine phenomena from the coupling. In particular some of these effects are expected to be similar to those occurring with thin sheets of liquid [16–18]. The present paper contains a discussion of such a possibility.

Thermal gradients in a floating layer may be caused by internal heating or cooling, induced by exo- or endothermal chemical reactions respectively. As in our previous publication [19] (hereafter called I) we consider a layer with a heated or cooled permeable divider (partition). An obvious possibility for an experimental realization exists with low/microgravity conditions in a spacecraft or even on Earth where, with suitably thin sheets, the divider may be used as a grid catalyst for possible exo- or endothermic chemical reactions. Thermal and hydrodynamic properties of permeable dividers and their influence on convective stability were treated earlier in theoretical and experimental studies on Bénard–Rayleigh buoyancy-driven convection [20–22]. An infinitely large hydrodynamic resistance of the divider reduces to the case of a two-layer system divided by a solid boundary. The opposite case of a heated divider with vanishingly small resistance may be realized by heating with a laser sheet. With respect to the results reported in I the new phenomena described here originate in the earlier-mentioned surface deformability and the eventual interaction between the two deformable surfaces because of the mechanical and/or thermal communication allowed by the divider. In Section 2 we state and solve the problem for monotonic instability. In Section 3 we formulate the problem of overstability. Section 4 contains results for large divider resistances. Section 5 contains results for the opposite case of small divider resistances when there is interaction between the disturbances at both surfaces, hence lead-

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NOMENCLATURE

h	half liquid layer depth
k	horizontal Fourier mode
M	Marangoni number $M = \gamma\Theta h/\eta\chi$
Pr	Prandtl number $Pr = \nu/\chi$
Ca	(inverse) capillary or crispation number $Ca = \sigma_0 h/\eta\nu$
p	pressure
T	temperature
T_0	temperature in the quiescent state
$\mathbf{v}$	velocity vector
v_x, v_z	horizontal and vertical velocity components
v	dimensionless amplitude of the vertical velocity component
C_i	coefficient of general solution
t	$= \tanh(k)$
x, z	horizontal and vertical coordinates.

Greek symbols

θ, Θ	dimensionless temperature disturbance and dimensional reference temperature
χ	liquid heat diffusivity
Δ	Laplacian operator
σ	surface tension; equation of state: $\sigma = \sigma_0 - \gamma T$
γ	surface tension temperature coefficient
η	dynamic viscosity
ν	kinematic viscosity
ρ	density
ζ	dimensionless deviation of liquid surface from the quiescent state position
ξ	amplitude of ζ
α,	α_τ, α_n ($\hat{\alpha}_\tau$, $\hat{\alpha}_n$) dimensionless divider/partition resistances: generic, tangential/parallel and transverse/normal, respectively (quantities with hat ^ have dimension).

ing to new regions of instability which extends the known results for isothermal thin liquid sheets to non-isothermal layers [16–18].

2. FORMULATION OF THE PROBLEM AND RESULTS FOR MONOTONIC INSTABILITY LEADING TO STATIONARY CONVECTION

Let us consider a floating thin liquid layer ($-h < z < h$) with the two outer surfaces free, open to air (see Fig. 1). At $z = 0$, i.e. midway between the two open boundaries, we insert a pervious/permeable divider which on average is uniformly heated or cooled. Disregarding gravitational effects we shall consider the result of the Marangoni effect acting at the two opposite surface boundaries of the liquid layer. In the quiescent state the temperature profile is assumed to be initially linear

$$T_0 = \Theta(1 - z/h) \quad \text{at} \quad z > 0$$

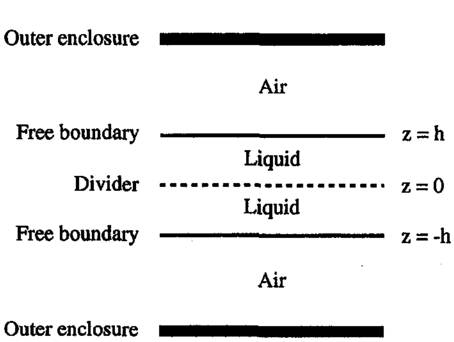

Fig. 1. A sketch of the geometry of the problem.

and

$$T_0 = \Theta(1 + z/h) \quad \text{at} \quad z < 0 \tag{1}$$

with Θ defined by the power at the divider as a heat source or sink. We have set the zero reference value at the outer boundary of the liquid layer. Note that although we start with isothermal boundary conditions (b.c.) we assume that due to the outer ambience air having a largely lower heat diffusivity than the liquid the actual b.c. for disturbances is adiabatic. Note also that we allow for surface deformation. The temperature of the divider Θ may be either positive or negative.

Let T, p, v denote infinitesimal temperature, pressure and velocity disturbances respectively. They are governed by the linearized form of the Navier–Stokes, continuity and heat transport equations:

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\nabla p + \nu \Delta v, \quad \operatorname{div} v = 0, \quad \frac{\partial T}{\partial t} + v \nabla T_0 = \chi \Delta T \tag{2}$$

where ρ is the density, ν is the kinematic viscosity and χ is the heat diffusivity of the liquid.

Assume that transverse displacements of the free liquid surfaces ζ are small quantities, i.e. for a linear approximation the b.c. may be applied at $z = \pm h$. At the open boundaries with a linear temperature dependence of the surface tension $\sigma = \sigma_0 - \gamma T$, we assume that air plays no active role, hence

$$z = \pm h: \quad v_z = \partial\zeta/\partial t, \quad p \pm \sigma_0 \frac{\partial^2 \zeta}{\partial x^2} = 2\rho\nu \frac{\partial v_z}{\partial z},$$

$$-\gamma\left(\frac{\partial T}{\partial x}+\frac{\partial T_0}{\partial z}\frac{\partial \zeta}{\partial x}\right)=\pm\rho\nu\left(-\frac{\partial v_x}{\partial z}+\frac{\partial v_z}{\partial x}\right) \quad (3)$$

where ζ is the deviation of either of the two layer surfaces from their level position at the quiescent state. The disturbed heat flux at both free surfaces is assumed such that

$$\partial T/\partial z(z=\pm h)=0 \quad (4)$$

hence there are adiabatic conditions leading to vanishing heat flux disturbances.

At the permeable divider, i.e. at level $z=0$ we assume continuity of the velocity:

$$v_x^+=v_x^-,\quad v_z^+=v_z^- \quad (5)$$

where the upper indices '+' and '−' denote values above and below the divider respectively. Furthermore, we assume that the through-flow obeys Stokes law, i.e. the tangential velocity component is proportional to the sum of the tangential stresses on either side of the partition, while the normal component is proportional to the pressure jump across it

$$v_x=\hat{\alpha}_\tau^{-1}\rho\nu(\partial v_x^+/\partial z-\partial v_x^-/\partial z),\quad v_z=-\hat{\alpha}_n^{-1}(p^+-p^-) \quad (6)$$

where $\hat{\alpha}_\tau$ and $\hat{\alpha}_n$ are phenomenological parameters that define the tangential/parallel and normal/transverse hydrodynamic divider resistances. Thermal conditions at the divider are such that disturbances of temperature and heat flux are continuous:

$$T^+=T^-,\quad \partial T^+/\partial z=\partial T^-/\partial z. \quad (7)$$

For a dimensionless description of the problem we use suitable units: distance, h; time, h^2/ν; velocity, χ/h; divider resistances, $\rho\nu/h$; and temperature, Θ. Then with these new 'scales', using the Fourier decomposition

$$v_z=v(z)\exp(-\lambda t+ikx),$$
$$T=\theta(z)\exp(-\lambda t+ikx),$$
$$\zeta=\xi\exp(-\lambda t+ikx)$$

where primes (′) denote z-derivatives. Restricting consideration to steady modes of instability, we have

$$v''''-2k^2v''+k^4v=0,\quad \theta''-k^2\theta\pm v=0 \quad (8)$$

with b.c.

$z=\pm 1$:

$$v=0,\quad v'''-3k^2v'\mp Ca\,Pr\,k^4\xi=0,$$
$$v''+k^2v+k^2M(\pm\theta-\xi)=0,\quad \theta'=0 \quad (9)$$

$z=0$:

$$v^+=v^-,\quad v^{+\prime}=v^{-\prime},\quad v^{+\prime\prime\prime}-v^{-\prime\prime\prime}=-\alpha_n k^2 v,$$
$$v^{+\prime\prime}-v^{-\prime\prime}=\alpha_\tau v'\theta^+=\theta^-,\quad \theta^{+\prime}=\theta^{-\prime}. \quad (10)$$

Five dimensionless groups have been introduced: the (inverse) capillary or crispation number, $Ca=\sigma_0 h/\rho\nu^2$, the Prandtl number, $Pr=\nu/\chi$, the Marangoni number, $M=\gamma\Theta h/\rho\nu\chi$, $\alpha_n=\hat{\alpha}_n h/\rho\nu$ and $\alpha_\tau=\hat{\alpha}_\tau h/\rho\nu$.

Solutions of equations (8)–(10) naturally separate into even $\{v(-z)=v(z),\ \theta(-z)=-\theta(z)\}$ and odd $\{v(-z)=-v(z),\ \theta(-z)=\theta(z)\}$ modes. It is convenient to construct the solution for the region $0<z\leqslant 1$ and to suitably re-define the b.c. at $z=0^+$. We obtain:

(i) for the even solution at $z=0^+$:

$$v'=0,\quad v'''+0.5k^2\alpha_n v=0,\quad \theta=0 \quad (11)$$

(ii) for the odd solution at $z=0^+$:

$$v=0,\quad v''-0.5\alpha_\tau v'=0,\quad \theta'=0. \quad (12)$$

Hence the even solution depends only on the transverse resistance α_n, while the odd mode depends on the tangential one α_τ.

The general solution of both problems can be written in the region $z>0$ as

$$v=C_1\sinh kz+C_2\cosh kz+C_3 z\cosh kz+C_4 z\sinh kz$$
$$\theta=C_5\sinh kz+C_6\cosh kz+(C_4/4k^2-C_2/2k)$$
$$z\sinh kz+(C_3/4k^2-C_1/2k)z\cosh kz$$
$$-(C_3\sinh kz+C_4\cosh kz)z^2/4k \quad (13)$$

where C_i are constants determined by the boundary conditions.

For the even solution the solvability condition of the system (9), (11) with (13) yields the neutral stability curve, i.e. provides the Marangoni number as a function of wave number and the dimensionless groups of the problem. We have

$$M=[32k^2+8\alpha_n(k^2t^2+kt-k^2)]/[4(k^2-2kt$$
$$+t^2-k^2t^2+2kt^3)+\alpha_n(t^3/k-k^2+k^2t^2)$$
$$+(Ca\,Pr)^{-1}8k^2(t^2-1)(4-\alpha_n)] \quad (14)$$

with $t=\tanh k$. Clearly for a plane liquid surface ($Ca\,Pr\to\infty$) equation (14) reduces to equation (11) of Ref. [1].

Asymptotically we have:

1(a) $k\to 0$:

$$M=[480+80\alpha_n k^2]/[60k^2+\alpha_n k^4$$
$$+120(Ca\,Pr)^{-1}(\alpha_n-4+4k^2)] \quad (15)$$

and for $\alpha_n>4$ the onset of instability occurs at $M>0$ (heated divider). At $k=0$ the Marangoni number is equal to $4Ca\,Pr/(\alpha_n-4)$. Thus for $\alpha_n<4$ there is a region of small k where instability exists at $M<0$ (cooled divider or heated ambient air). For large values of the combination $Ca\,Pr$ the region is $0\leqslant k^2<2(4-\alpha_n)/Ca\,Pr$ with $M_c=-4Ca\,Pr/(4-\alpha_n)$.

1(b) $k\to\infty$: $M=8k^2$, i.e. the instability appears only for a heated divider.

Typical neutral stability curves for the even mode

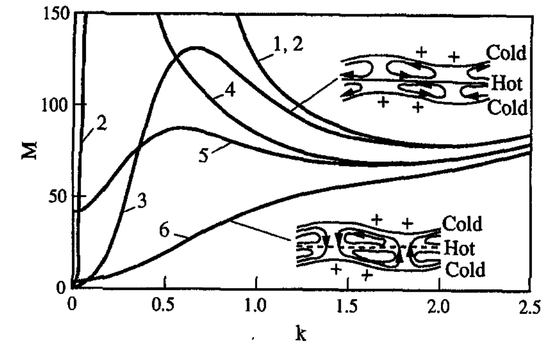

Fig. 2. Neutral curves for even monotonic disturbances at large divider resistances and different values of the combination $Ca\ Pr$: 1—$\alpha_n = \infty$, $Ca\ Pr = \infty$; 2—∞, 10^5; 3—∞, 10^3; 4—100, ∞; 5—100, 10^3; 6—100, 100.

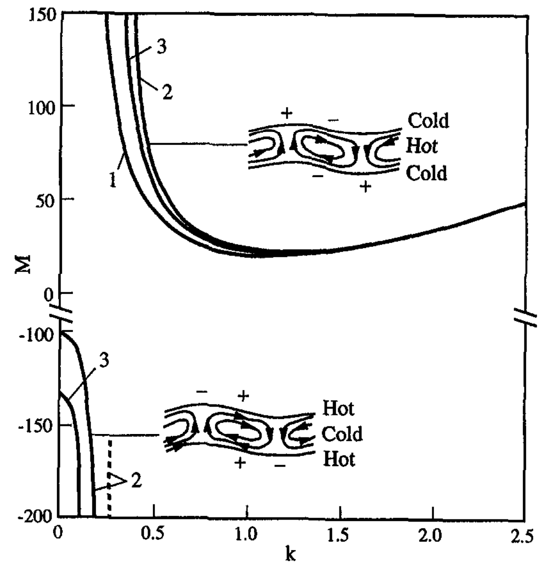

Fig. 3. Neutral curves for even monotonic disturbances at low divider resistances and different values of $Ca\ Pr$: 1—$\alpha_n = 0$, $Ca\ Pr = \infty$; 2—0, 100; 3—1, 100.

in the case $\alpha_n > 4$ are shown in Fig. 2. Curve 1 corresponds to an impervious divider ($\alpha_n \to \infty$) and an undeformable plane surface layer ($Ca = \infty$). The curve has only one minimum at $k_c = 2.0$ with $M_c = 80$ (as in Ref. [1]). For a deformable surface (finite values of Ca) and an impervious divider the lowest threshold instability appears at $k = 0$ (curves 2 and 3). For finite values of the resistance the latter instability vanishes and the critical Marangoni number increases as α_n is decreased. In some range of α_n and $Ca\ Pr$ the neutral curves possess two minima (e.g. curve 5). Further decreasing the resistance makes the disturbances with finite wave number become the most dangerous. For $\alpha_n = 4$ the vertical axis is the asymptote of the neutral curve at $k = 0$. The inset in Fig. 2 illustrates qualitatively the shape of critical disturbances and the distortion pattern for the free surfaces. It is readily seen that bulging of the fluid surface is caused by the fluid inflow from the regions with higher surface temperatures. The index '+' denotes the regions of the surface in which the Marangoni effect sustains the disturbances. In the valleys, the heated fluid is carried by the velocity disturbance to the surface, hence Pearson's [1] result for instability of a layer with a flat open surface. Mismatch in the slope of the layer surface with respect to the isotherms of the basic temperature field provides the second mechanism sustaining disturbances [13]. In contrast to Pearson's this mechanism operates efficiently in the region of long waves and for the case with an impervious divider, contributes to the onset of the lowest threshold instability at $k = 0$.

For $\alpha_n < 4$ the asymptote is the straight line $k_a \simeq [2(4-\alpha_n)/Ca\ Pr]^{1/2}$ and a new instability range appears when the layer is heated from outside or there is a cooled divider. Typical neutral stability curves for this case are shown in Fig. 3. Curve 1 represents the case of a layer with undeformable boundaries and a permeable divider. As in the case $\alpha_n = \infty$ this curve is asymptotic to the vertical axis at $k = 0$. Curves 2 and 3 ($Ca\ Pr = 100$, $\alpha_n = 0$ and 1 respectively) have discontinuities at $k = k_a$ and illustrate the presence of instability for both ways of heating.

Instability in the short-wavelength region is related to Pearson's mechanism [1] and occurs with a heated divider. A typical flow pattern is depicted in the upper inset of Fig. 3. Contrary to the cases displayed in Fig. 2, distortion of the fluid surface in the layer with the permeable divider should be attributed to transverse flows. The Marangoni effect at the inclined section of the free surface slows down the initiated motion (this point is marked by index '−') and suppresses it in the region of long waves. However, with the layer heated from outside this mechanism leads to a long-wavelength instability, as shown at the bottom of Fig. 3. Here, as the depicted flow pattern shows, Pearson's mechanism prevents the growth of disturbances. Heating from outside or with a cooled divider, the disturbances with $k = 0$ are the most dangerous. Since a new mechanism of instability makes itself evident only at low divider resistances and within the long-wavelength range, it seems to be related to the interaction of disturbances occurring at the two opposite free surfaces.

For the odd solutions the solvability condition yields

$$M = 8[4k^2t^3 + \alpha_\tau(t^3k^2 + t^2k - k^2t)]/[4(t^3 - k^2t + k^2t^3) + \alpha_\tau(t^2/k - 2t + 2t^3 + k - kt^2 - k^2t + k^2t^3) + 8k^2(Ca\,Pr)^{-1}(1-t^2)(4+\alpha_\tau)] \quad (16)$$

which asymptotically provides:

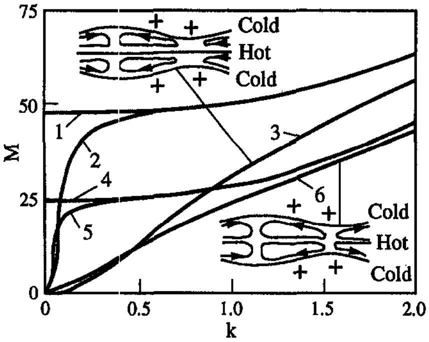

Fig. 4. Neutral curves for odd monotonic disturbances at different values of α_τ and $Ca\ Pr$: 1—$\alpha_\tau = \infty$, $Ca\ Pr = \infty$; 2—∞, 10^4; 3—∞, 10^2; 4—0, ∞; 5—0, 10^4; 6—0, 10^2.

2(a) $k \to 0$:

$$M = [288(1-k^2)+16\alpha_\tau(3-4k^2)]/[12(1-2k^2) + \alpha_\tau(1-4k^2)+24(k^2 Ca\,Pr)^{-1} (3-4k^2)(4+\alpha_\tau)].$$

2(b) $k \to \infty$:

$$M \approx 8k^2. \qquad (17)$$

Instability appears only when we have a heated divider. Typical neutral curves for the odd mode are shown in Fig. 4. For undeformable surfaces ($Ca = \infty$) the instability occurs with $k = 0$ at finite critical Marangoni numbers (curves 1, 4). With a deformable surface the odd mode always yields the lowest instability threshold ($M = 0$ at $k = 0$). This is typical of microgravity conditions. Obviously with finite gravitational acceleration our b.c. [in particular equation (3)] need to be augmented with the hydrostatic contribution and as buoyancy may play a non-negligible role due care must be given to the possible Rayleigh–Taylor instability of the lower surface.

The inset of Fig. 4 depicts the convective flow in the case of a solid boundary (the upper inset) and a divider with $\alpha_\tau = 0$. The solid divider suppresses fluid motion in the middle of the layer and moves the convective rolls towards the free surface. This provides stability of the layer to disturbances of finite k as the divider resistance is increased. As can be seen from the figure, the convective flow is sustained by the two earlier mentioned instability mechanisms. The lowest threshold for instability is related to motion generated at the part of the surface not aligned with the isotherms of the basic temperature field.

3. OVERSTABILITY

Until now we have discussed the possibility of instability in the form of stationary convection. Now we turn our attention to oscillatory modes of convection. The amplitudes of oscillatory disturbances $v(z)$, $\theta(z)$ and ξ are governed by the equations

$$v'''' - 2k^2v'' + k^4v + \lambda(v'' - k^2v) = 0 \qquad (18a)$$

$$\theta'' - k^2\theta \pm v + \lambda Pr\theta = 0 \qquad (18b)$$

with b.c. on the free surfaces

$$z = \pm 1: \quad \lambda Cv''' - 3k^2v') + \lambda^2 v' \pm Cak^4 v = 0 \qquad (19a)$$

$$\lambda(v''+k^2v) \pm M\lambda k^2\theta + k^2MPr^{-1}v = 0, \quad \theta' = 0, \quad \xi = -Prv/\lambda \qquad (19b)$$

and b.c. (11) at the divider. We consider neutral disturbances with $\lambda = \mathrm{i}\omega$, where the non-vanishing ω provides the expected frequency of overstable modes.

The solutions of equations (18) with b.c. (19) are also separated into even and odd modes, with b.c. at $z = 0^+$, (11), (12). For both modes, the general solution, when $Pr \neq 0$ and $Pr \neq 1$, can be written for the region $z > 0$ as

$$v = C_1\exp(kz) + C_2\exp(-kz) + C_3\exp(\nu z) + C_4\exp(-\nu z)$$

$$\theta = -(C_1\exp(kz)+C_2\exp(-kz))/\mathrm{i}\omega Pr - (C_3\exp(\nu z)+C_4\exp(-\nu z))/\mathrm{i}\omega(Pr-1) + C_5\exp(\beta z)+C_6\exp(-\beta z), \qquad (20)$$

where $\nu = (k^2 - \mathrm{i}\omega)^{1/2}$, $\beta = (k^2 - \mathrm{i}\omega Pr)^{1/2}$. The solvability condition of the system for the coefficients C_i defines the overstable neutral curves, hence providing the corresponding frequency ω and Marangoni number. We have

$$\mathrm{Im}\,(\Delta_1(\omega)/\Delta(\omega)) = 0 \qquad (21)$$

while $M = Re(\Delta_1/\Delta)$, where Δ_1 and Δ for the even mode are

$$\Delta_1,\Delta = \det\begin{Bmatrix} A+ & A- & B+ & B- & 0 & 0 \\ C+ & C- & D+ & D- & 0 & 0 \\ E+ & E- & F+ & F- & G+ & G- \\ I+ & I- & J+ & J- & 0 & 0 \\ K+ & K- & L+ & L- & M+ & M- \\ N+ & N- & O+ & O- & P+ & P- \end{Bmatrix}$$

For Δ_1 and even mode

$$A\pm = \pm k, \quad B = \pm\nu, \quad C\pm = \pm k^3 + 0.5k^2\alpha_n,$$

$$D\pm = \pm\nu^3 + 0.5k^2\alpha_n, \quad E\pm = 1/\mathrm{i}\omega Pr,$$

$$F\pm = 1/\mathrm{i}\omega(Pr-1), \quad G\pm = -1,$$

$$I\pm = (\pm\omega^2 k \pm 2k^3\mathrm{i}\omega - Ca\,k^4)\exp(\pm k),$$

$$J\pm = (\pm\omega^2\nu \pm \mathrm{i}\omega\nu(3k^2-\nu^2) - Ca\,k^4)\exp(\pm\nu),$$

$$K\pm = 2\mathrm{i}\omega k^2\exp(\pm k),$$

$$L\pm = \mathrm{i}\omega(\nu^2+k^2)\exp(\pm\nu),$$

$$M\pm = 0, \quad N\pm = \mp k\exp(\pm k)/B_1,$$

$$O\pm = \mp\nu\exp(\pm\nu)/B_2, \quad P\pm = \pm\beta\exp(\pm\beta).$$

For Δ the same elements of the determinant

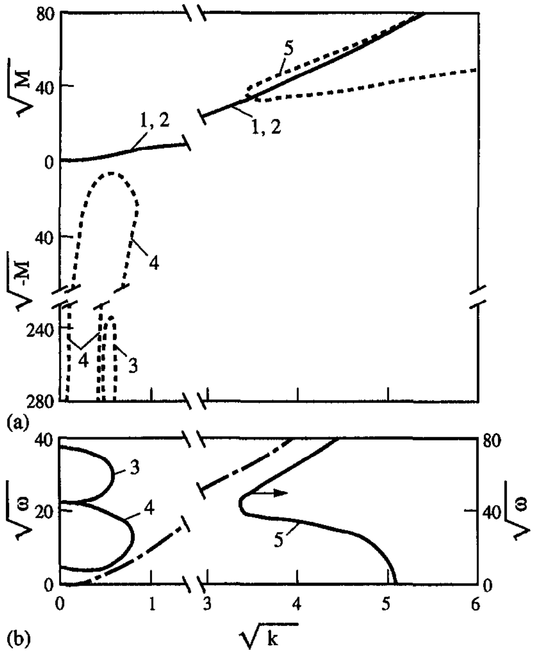

Fig. 5. Neutral curves (a) and frequencies (b) of neutral disturbances for a solid divider and $Pr = 0.01$, $Ca = 10^4$. The even and odd numbers refer, respectively, to even and odd disturbances. The dotted lines depict overstable neutral curves.

must be replaced by $K\pm = 0$, $L\pm = k^2(1/Pr-1/(Pr-1))\exp(\pm v)$, $M\pm = i\omega k^2\exp(\pm\beta)$.

For the odd mode some elements of Δ_1 and Δ must be replaced by e.g. $A\pm = B\pm = 1$, $C\pm = k^2 0.5k^3\alpha_\tau$, $D\pm = v^2 \mp k^2 v\alpha_\tau$, $E\pm = \pm k/i\omega Pr$, $F\pm = \pm v/i\omega(Pr-1)$, $G\pm = \mp\beta$.

4. OSCILLATORY INSTABILITY IN THE CASE OF AN IMPERVIOUS DIVIDER

An impervious divider separates the layer into two halves with vanishing hydrodynamic interaction. Mathematically the problem for the even mode is equivalent to that of a plane layer with a solid isothermal support, while the odd mode problem reduces to the case of a solid boundary with zero heat flux for disturbances. The difference in the behavior of even and odd disturbances should be most evident in the long wavelength region, whereas in the region of short waves it is negligible. The onset of thermocapillary instability in a plane layer with one free boundary [3, 5–7, 12] can be easily observed using small Prandtl number liquids. For illustration let us consider the case of a solid divider for a layer with fixed (inverse) capillary number $Ca = 10^4$, which is typical for layers of melted semiconductors.

Variations of the critical Marangoni number and the frequency of (neutral) oscillations with the wave number k are plotted in Fig. 5 for $Pr = 0.01$. The solid line at the top of the figure provides the neutral curve for monotonic disturbances discussed in Section 2. At given parameter values the curves describing even and odd disturbances coincide on the scale used in our plot.

The neutral curves for oscillatory disturbances are depicted by broken lines. The even and odd numbers of curves correspond, respectively, to even and odd modes of disturbances. In the region of long waves (small k) oscillatory instability occurs under external heating of the free surface or with a cooled divider. The dispersion relation of these waves differs significantly from the Laplace–Kelvin law ($\omega^2 = Ca\, k^3\tanh k$) for capillary waves in a plane layer of isothermal fluid (dashed–broken line in the figure). The frequency of oscillation of the observed waves remains finite at low k. These waves are sustained by the outward-directed temperature gradient of the fluid. In the following, these waves will be called 'thermocapillary' waves.

The onset of thermocapillary waves in the field of gravity was described in Takashima's work [3, see also 5–7] for liquid layers with one solid isothermal boundary. As discussed earlier, the discrepancy of neutral curves and frequencies for solutions with different symmetry is caused by disparity of temperature conditions imposed at the divider. The most dangerous is the even mode (curve 4), involving non-zero heat flux through the divider. In the region of short waves ($k > 12$) the oscillations are induced by internal heating of the fluid (curve 5 in Fig. 5). The existence of such waves in the fluid layer with a solid boundary was first reported in [12] and later in [15; see also 7, 23]. The dispersion relation for the waves at the lower branch is close to $\omega^2 = Ca\, k^3$. We have the excitation of capillary waves by the Marangoni thermocapillary effect. Because of their short length, these waves are localized close to the surface and are insensitive to the b.c. at the divider. Referring to Fig. 5, the frequency at the upper branch of the neutral curve 5 decreases with increasing wave number and goes to zero at $k = 26$ where the neutral curve of the oscillatory instability merges with the neutral curve for monotonic disturbances (curves 1,2). With increase of the Prandtl number, i.e. increased contribution of viscous dissipation or increased life-time of thermal disturbances, the region of short capillary waves is shifted towards still shorter waves. Apart from this, one may observe generation of capillary waves in the intermediate range of the wave numbers ($k \sim 1$) under external heating of the fluid surface. This region is adjacent to the region of the long-wavelength thermo-capillary instability.

Figure 6 gives the neutral curves and frequencies of neutral oscillations at $Pr = 0.1$ where the representation of curves is identical to that of Fig. 5. The neutral curves and frequencies of even and odd disturbances as depicted in Figs. 5 and 6 for $k > 1$ are essentially the same. Thus in the layer with a solid impervious divider, with $Pr = 0.1$, we may distinguish between three regions of excitable waves: ther-

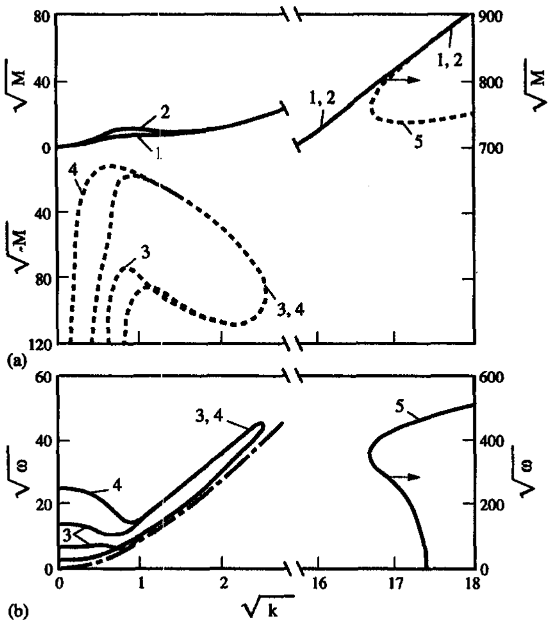

Fig. 6. Neutral curves (a) and frequencies (b) of neutral disturbances for a solid divider and $Pr = 0.1$, $Ca = 10^4$. The even and odd numbers refer, respectively, to even and odd disturbances. The dotted lines are taken for neutral curves of oscillatory disturbances.

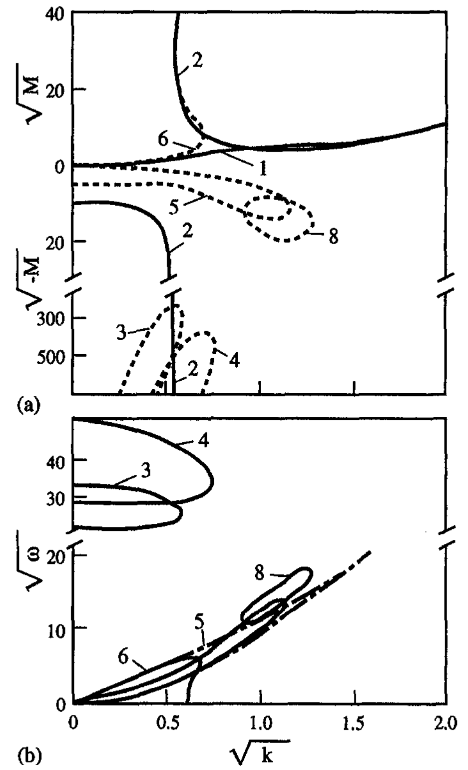

Fig. 7. Neutral curves (a) and frequencies (b) of neutral disturbances for a 'transparent' divider ($\alpha_n = 0$ and $\alpha_\tau = 0$). Symbols and parameters are identical to those used in Fig. 5.

mocapillary waves ($0 \leqslant k \leqslant 0.5$) and capillary waves ($0.5 < k < 6.3$), initiated by heating from outside, say, and short capillary waves ($k > 276$) sustained by heating at the divider of the liquid layer. The difference between the layer with a solid divider and that with a solid continuous boundary becomes evident only in the region of long waves, in which one may expect the appearance of waves described by various thermal conditions on the solid boundary.

5. OSCILLATORY INSTABILITY IN THE CASE OF A TRANSPARENT DIVIDER

Let us now turn our attention to the case when the divider exerts none or negligible mechanical action on the fluid motion. Hence the divider serves only as a heat source or heat sink. The former situation may occur when heating the fluid with a thin laser sheet or it may simply be considered as a simplified model of a more complicated mathematical problem on thermoconvective instability caused by internal heat sources.

In the region of short waves ($k > 10$) the situation is very much like the case of excited surface waves with a solid divider, since such disturbances really do not reach the divider.

In the region of intermediate and long waves the waves generated on opposite layer surfaces are expected to undergo strong hydrodynamic interaction. For a thin isothermal layer, waves may be classified into two groups [16–18]: antisymmetrical/flexural waves (even mode) with dispersion relation $\omega^2 = Ca\ k^3 c\tanh k$ and symmetrical/squeezing waves (odd mode) with dispersion relation $\omega^2 = Ca\, k^3 \tanh k$. Instability of the non-isothermal fluid layer with respect to oscillatory disturbances involving dispersion relations, closely approximating the earlier mentioned Laplace–Kelvin law, should be treated as capillary waves genuinely excited by the Marangoni effect.

The neutral curves for disturbances in the layer with such 'transparent' partition are plotted in Fig. 7 for $Pr = 0.01$ and $Ca = 10^4$. The curves describing even and odd disturbances are labelled, respectively, by the even and odd numbers. The solid lines at the top of the figure represent the neutral curve for monotonic disturbances. As already stated the neutral curve for even disturbances at low values of the divider resistance has two branches corresponding to a long-wavelength region at negative Marangoni numbers and to a short-wavelength region at positive values respectively. These curves are labelled with the number 2.

The even and odd thermocapillary waves (curves 3,4) with non-zero frequency at $k = 0$ appear in the region of long waves at sufficiently large negative Marangoni numbers (external heating). On the other hand, this region also involves the lowest threshold instability with respect to capillary waves (curves 5,6). The dispersion curves for these waves are actually consistent with the curves for isothermal capillary

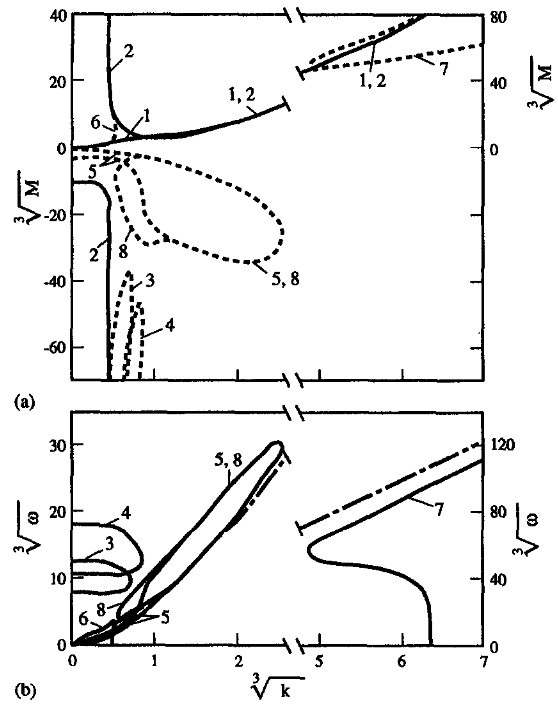

Fig. 8. Neutral curves (a) and frequencies (b) of neutral disturbances for a 'transparent' divider ($\alpha_n = 0$ and $\alpha_\tau = 0$). $Pr = 0.01$, $Ca = 10^5$. Symbols are identical to those of Fig. 5.

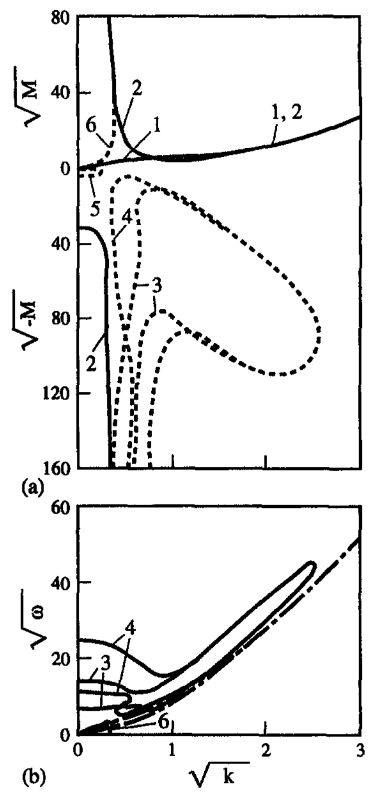

Fig. 9. Neutral curves (a) and frequencies (b) of neutral disturbances for a 'transparent' divider ($\alpha_n = 0$ and $\alpha_\tau = 0$). $Pr = 0.1$, $Ca = 10^5$. Symbols are identical to those of Fig. 5.

waves (dash–dotted lines). Note that the symmetrical wave (curve 5) occurs in a rather narrow range of Marangoni numbers, when the fluid layer is heated from outside or there is a cooled divider. In contrast, the antisymmetrical waves are observed at all positive Marangoni numbers (above the neutral curve) and nearly the same wavenumbers k, corresponding to the onset of monotonic instability with respect to the even mode under external heating. The upper branch of curve 6 ends at the curve of monotonic instability, labelled 2.

With a fluid layer heated from outside there is also a region of antisymmetrical capillary waves with wave number $k \sim 1$, which is limited by the Marangoni and wave numbers (the inner region of curve 8). At $Pr = 0.01$ this region exists only at fairly large (inverse) Capillary numbers ($Ca > 8 \times 10^3$) i.e. at relatively small surface deformation. As the (inverse) capillary number increases the region of excitable capillary waves extends as seen in Fig. 8, which shows neutral curves for $Ca = 10^5$. Note that the change in the scale relative to the previous figures allows all the neutral curves to be accommodated in a single figure.

It should be emphasized that at $Pr = 0.01$ the capillary and thermocapillary waves exist in separate regions. The situation is different with large Prandtl numbers. Neutral curves for the fluid with $Pr = 0.1$ and low divider resistance are displayed in Fig. 9. Here a radically new feature can be observed, namely the appearance of a wide region of sustained capillary waves in the intermediate wave number range when heating the layer from outside or when there is a cooled divider ($M < 0$). This region transfers constantly to the region of long thermocapillary waves. Note however, that the lowest threshold instability with respect to long capillary waves governed by the Laplace–Kelvin relation is still valid.

6. CONCLUSIONS

In the present work we have analysed various aspects of the Marangoni instability of a floating liquid layer with two free surfaces and a permeable divider located in the mid-plane that serves as a heat source or heat sink. The available temperature distribution endows both fluid surfaces with equal possibilities for instability, hence the interaction of disturbances generated at the two opposite layer surfaces. For monotonic disturbances the parameters are the Marangoni number and the combination product of Prandtl and (inverse) capillary numbers, whereas for oscillatory disturbances these three parameters appear separately. Symmetry in the problem reduces all disturbances to even/flexural/antisymmetrical and odd/squeezing/symmetrical disturbances.

The stability analysis of the layer for monotonic disturbances carried out in Section 2 shows that for a high-resistance divider the situation is similar to the case with a solid boundary. For even disturbances the boundary conditions at the divider are identical to those for an isothermal solid surface, whereas for odd disturbances the heat flux is fixed. Even disturbances show two well-defined instability mechanisms: Pearson's mechanism [1] with wavenumber $k \sim 2$ and a long-wavelength instability related to the existence of the temperature gradient along the surface because of its bending, and resulting in the lowest instability threshold at finite (inverse) capillary number and infinite divider resistance. For odd disturbances the second mechanism at finite (inverse) capillary numbers also leads to the lowest long-wavelength instability threshold.

The case of a low-resistance divider, characterized by a strong hydrodynamic interaction of the long-wavelength disturbances at both sides of the divider, may be of considerable interest when dealing with the onset of long-wavelength monotonic instability for symmetrical waves when the divider is used as a cooling device. This instability exists only for $\alpha_n < 4$.

For oscillatory disturbances, leading to surface waves, the case of a permeable divider also shows the essential difference with the case of a solid continuous boundary. The latter is equivalent to two stability problems for a plane layer with the solid continuous boundary subject to opposite limiting thermal conditions. The oscillations were examined in the region of small Prandtl numbers and large (inverse) capillary numbers, when the waves along the fluid surface have a relatively small viscous dissipation. Comparison of the dispersion relations with that for the capillary waves in isothermal fluid allows us to distinguish between two types of oscillatory instability in the non-isothermal case: capillary waves sustained by the Marangoni effect and long thermocapillary waves with non-vanishing frequency at $k \to 0$. The former, thermocapillary waves always occur for fluid layers heated from outside, while capillary waves may be sustained with either way of heating. In all cases the capillary waves appear at sufficiently large values of k when there is a heated divider. For a divider of high resistance, as well as in the layer with a solid boundary, the region of small k is found to be stable to capillary waves. For a permeable divider the very long capillary waves with $k \to 0$ correspond to the lowest threshold instability of the layer both when heating from outside and inside, i.e. for both negative and positive Marangoni numbers respectively.

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